# Improvement of Gm (1,1) Model Based on Particle Swarm Optimization Algorithm--Model Performance Analysis Based on Epu Data

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**Abstract:** Based on the analysis of GM (1,1) model, through theoretical derivation: (1) the first data has no effect on the predicted value, (2) when the same increment is added to each data of the original sequence, the predicted value will change. Therefore, it is proposed to insert new data before the first data of the sequence, add the same increment at each position, and use particle swarm optimization algorithm to get the best increment. By comparing the errors, this method can improve the prediction accuracy of the model.

## 1. Introduction

Grey system prediction method can effectively deal with data with few samples, and has less requirements for sample mean and variance. Since Deng Julong put forward it, it has attracted the attention of researchers, and has made a lot of pioneering work in the fields of earthquake prediction, meteorological prediction, pest prediction and so on. With the deepening of research, catastrophe model, multivariate model, GM(1,1) and its extension model are introduced. By analyzing the properties of GM(1,1) model, this paper puts forward an improved calculation method to improve the accuracy.

## 2. Introduction of Gm (1,1) Model

It is called grey modeling that the sequence is built into a model compatible with differential, difference and approximate exponential law. The first-order GM model with n variables is recorded as GM(1,n), and the first-order GM model with 1 variable is recorded as GM(1,1). The equation is as follows:

$$\frac{dx}{dt}+ax=b \qquad (1)$$

$a$ and $b$ are constants and (1) is a general differential equation.

Operate AGO(accumulated generating operation) on sequence $x^{\{0\}}=\{x^{\{0\}}(1),x^{\{0\}}(2),\cdots,x^{\{0\}}(n)\}$, we can obtain sequence $x^{\{1\}}=AGOx^{\{0\}}$ and GM (1,1) grey differential equation (2):

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \qquad (2)$$

Let $\hat{a}$ be the parameter vector and $\hat{\boldsymbol{a}}=\begin{bmatrix}a\\b\end{bmatrix}$. Then the following results can be obtained by OLS method.

$$\hat{\boldsymbol{a}}=(\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{y}_N \qquad (3)$$

where

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$$\boldsymbol{B}=\begin{bmatrix} -\frac{1}{2}[x^{(1)}(1)+x^{(2)}(2)] & 1 \\ -\frac{1}{2}[x^{(1)}(2)+x^{(2)}(3)] & 1 \\ \vdots & \vdots \\ -\frac{1}{2}[x^{(1)}(n-1)+x^{(2)}(n)] & 1 \end{bmatrix} \tag{4}$$

and

$$\boldsymbol{y}_N=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{5}$$

Time response formula of GM(1,1) model as follows:

$$\begin{cases} \hat{x}^{(1)}(k+1)=\left(x^{(0)}(1)-\frac{a}{b}\right)e^{-ak}+\frac{a}{b} \\ \hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) \end{cases} \tag{6}$$

By IAGO (Inverse accumulated generating operation), we can obtain the prediction of original sequence:

$$\begin{cases} \hat{x}^{(0)}(k+1)=\left(x^{(0)}(1)-\frac{a}{b}\right)(1-e^{a})^{-a(k-1)} \\ \hat{x}^{(0)}(1)=x^{(0)}(1) \end{cases} \tag{7}$$

## 3. Analysis of Gm(1,1) Model

### 3.1 The First Data of the Sequence Has No Effect on the Prediction Result

Give sequence $Y^{(0)}=\{y^{(0)}(1),y^{(0)}(2),\cdots,y^{(0)}(1)\}=\{x^{(0)}(1)+e,x^{(0)}(2),\cdots,x^{(0)}(n)\}$. By equation (7), we can get:

$$\begin{cases} \hat{y}^{(0)}(k)=\left(x^{(0)}(1)+e-\frac{a_1}{b_1}\right)(1-e^{a_1})^{-a_1(k-1)} \\ \hat{y}^{(0)}(1)=x^{(0)}(1)+e \end{cases} \tag{8}$$

We can also get:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix}=(\boldsymbol{B}_1{}^T\boldsymbol{B}_1)^{-1}\boldsymbol{B}_1{}^T\boldsymbol{y}_N \tag{9}$$

and

$$\boldsymbol{y}_N=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \tag{10}$$

where

$$\boldsymbol{B}_1=\boldsymbol{B}\begin{bmatrix} 1 & 0 \\ -e & 1 \end{bmatrix}=\boldsymbol{BP},\ \ \boldsymbol{P}=\begin{bmatrix} 1 & 0 \\ -e & 1 \end{bmatrix} \tag{11}$$

Therefore

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = (\boldsymbol{B}_1{}^T \boldsymbol{B}_1)^{-1} \boldsymbol{B}_1{}^T \boldsymbol{y}_N = (\boldsymbol{P}^T \boldsymbol{B}^T \boldsymbol{B} \boldsymbol{P})^{-1} \boldsymbol{P}^T \boldsymbol{B}^T \boldsymbol{y}_N = \boldsymbol{P}^{-1} (\boldsymbol{B}^T \boldsymbol{B})^{-1} (\boldsymbol{P}^T)^{-1} \boldsymbol{P}^T \boldsymbol{B}^T \boldsymbol{y}_N$$
$$= \boldsymbol{P}^{-1} (\boldsymbol{B}^T \boldsymbol{B})^{-1} \boldsymbol{B}^T \boldsymbol{y}_N = \begin{bmatrix} 1 & 0 \\ e & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ ae+b \end{bmatrix} \tag{12}$$

and

$$\begin{cases} \hat{y}^{(0)}(k) = \left( x^{(0)}(1) - \frac{a}{b} \right) (1 - e^{a})^{-a(k-1)} \\ \hat{y}^{(0)}(1) = x^{(0)}(1) + e \end{cases} \tag{13}$$

Result 1: frome equation (7 and 13), the first data $x^{(0)}(1)$ of the sequence has no effect on the prediction result $\hat{x}^{(0)}(k)(except\ \hat{x}^{(0)}(1))$. The mathematical quality leds to the low utilization of the first data.

Corollary1: Therefore, this problem can be improved by inserting new data before the first data.

### 3.2 Adding an Increment to Each Value of the Original Data Sequence Will Affect the Prediction Results

Give sequence $Y^{(0)} = \{y^{(0)}(1), y^{(0)}(2), \cdots, y^{(0)}(1)\} = \{x^{(0)}(1)+e, x^{(0)}(2)+e, \cdots, x^{(0)}(n)+e\}$ . By equation (7), we can get:

$$\begin{cases} \hat{y}^{(0)}(k) = \left( x^{(0)}(1) + e - \frac{a_2}{b_2} \right) (1 - e^{a_2})^{-a_2(k-1)} \\ \hat{y}^{(0)}(1) = x^{(0)}(1) + e \end{cases} \tag{14}$$

and

$$\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = (\boldsymbol{B}_2{}^T \boldsymbol{B}_2)^{-1} \boldsymbol{B}_2{}^T y_2 \tag{15}$$

$$y_2 = \begin{bmatrix} x^{(0)}(2)+e \\ x^{(0)}(3)+e \\ \vdots \\ x^{(0)}(n)+e \end{bmatrix} \tag{16}$$

$$\boldsymbol{B}_2 = \boldsymbol{B} \text{ - } \boldsymbol{e} \begin{bmatrix} 3/2 & 0 \\ 5/2 & 0 \\ \vdots & \vdots \\ (2n-1)/2 & 0 \end{bmatrix} \tag{17}$$

Result 2: from equation (17), we can conclude that Increment *e* has an impact on prediction result ( $\hat{y}^{(0)}(k)$ ).

## 4. Improvement of Gm (1,1) Model

According to the second chapter (combine with Result 1 &2, Corollary 1), the improvement ideas of this section are as follows.

(1)Add increment $e_1$ for each data in the new sequence.

(2)The particle swarm optimization algorithm is used to find the optimal $e_1$.

(3)Insert new data $e_1$ before the first data, i.e., set the increment to the same number as the insertion value before the first data of the original sequence.

(4)Use new data for GM (1,1) prediction.

### 4.1 Introduction and Operation Steps of Particle Swarm Optimization Algorithm

Particle swarm optimization algorithm is to simulate the feeding process of birds in the air. The position of a bird in the search space can be regarded as the solution of the problem in particle swarm optimization algorithm, which is called particle swarm optimization.

Suppose the velocity of the $k$ particle is $v_k$ and the position is $x_k$. The position of the optimal solution found by the $k$ particle is $p_{best\text{-}k}$. The position of the optimal solution found by the whole particle swarm is $g_{best\text{-}k}$. Let the inertia weight $c_0$ be a random number between 0 and 1. Let cognitive weight $c_1$ and social weight $c_2$ be random numbers between 0 and 2.

Then let

$$\begin{aligned} v_{k+1} &= c_0 v_k + c_1(p_{best-k} - x_k) + c_2(g_{best-k} - x_k) \\ x_{k+1} &= x_k + v_{k+1} \end{aligned} \quad (18)$$

The speed $v$ is limited as follows. Let

$$\begin{aligned} &if \quad v_k > v_{\max}, \quad v_k = v_{\max} \\ &if \quad v_k < -v_{\min}, \quad v_k = -v_{\min} \end{aligned} \quad (19)$$

The operation steps are as follows.

(1)Set the number of particles of the algorithm to $n$, and randomly generate $n$ initial velocities and initial solutions.

(2)Use equation (19) to calculate the new position of each particle.

(3)If the current number of iterations is less than the maximum number of iterations, proceed to the next step, otherwise stop.

(4)Sum the squares of the errors according to the new position of each particle. For each particle, if the sum of squares of the errors of the particles is smaller than the sum of squares of the errors at the original position, the position of the optimal solution of the individual particle ($p_{best\text{-}k}$) is updated.

(5)According to the position of the optimal solution of each particle ($p_{best\text{-}k}$.), the global extremum ($g_{best\text{-}k}$) is found.

(6)Recalculate the new speed and position by using equation (18).

(7)Number of iterations plus 1. Proceed to step 3.

### 4.2 Model Performance Analysis Based on Economic Policy Uncertainty (Epu) Index

The monthly data of EPU from January to October 2021 are selected. The data from Jan. to Jul. are used for model fitting, and the data from Aug. to Oct. are used for prediction accuracy test.

Table 1 EPU Data from Jan. to Oct. 2021

| Date | EPU |
|---|---|
| 202101 | 665.31 |
| 202102 | 565.40 |
| 202103 | 493.90 |
| 202104 | 488.23 |
| 202105 | 492.68 |
| 202106 | 413.07 |
| 202107 | 505.58 |
| 202108 | 592.80 |
| 202109 | 358.36 |
| 202110 | 398.36 |

The optimal increment (insertion value) obtained by particle swarm optimization algorithm is 15.65.

In order to further analyse the efficiency of the improved model, GM (1,1) is used as the benchmark model. The error analysis results are shown in the table below.

Table 2 Result of Prediction by using GM(1,1) and Improved Model Based on Particle Swarm Optimization Algorithm

| Model | | GM(1,1) | | Improved Model | |
|---|---|---|---|---|---|
| Date | Actual | Predict | Relative Error | Predict | Relative Error |
| 202108 | 592.80 | 439.53 | 25.86% | 423.50 | 3.65% |

| | | | | | |
|---|---|---|---|---|---|
| 202109 | 358.36 | 425.52 | 18.74% | 400.47 | 5.89% |
| 202110 | 398.36 | 411.95 | 3.41% | 378.70 | 8.07% |
| Mean | | | 16.00% | | 5.87% |

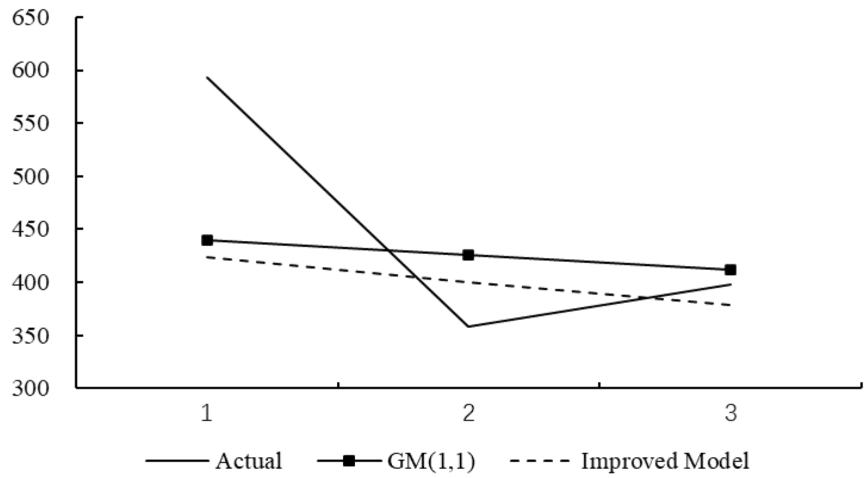


Fig.1 Line Chart of Prediction by Using Gm(1,1) and Improved Model

### 4.3 Conclusion

This paper proposes that the improved model has higher accuracy. Taking the prediction of EPU as an example, on average, it decreased from 16.00% to 5.87, a total decrease of about 10 percentage points. Therefore, the method of finding the optimal increment (also the insertion value before the first data of the original sequence) by particle swarm optimization algorithm can effectively improve the prediction accuracy. This method is simple in operation and calculation, and has a good application prospect.

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